Magnetic field dynamics in isolated neutron stars: insights from GRMHD simulations

Aurora Capobianco

Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena

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Collaborators: William Cook, Sebastiano Bernuzzi, Brynmor Haskell

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Motivation

- **Neutron stars** (NS) host extreme **magnetic fields** up to 10^{16}G: impact on NS structure, dynamics and evolution.
- Effects also on **observables**: EM radiation, outbursts, FRBs and gravitational waves.
- **Long-term equilibrium** configuration unclear.
- Pulsar observations point towards a large-scale **dipolar** field [Chung & Melatos 2011].
- **Purely poloidal** fields shown to be **unstable** [Tayler 1957, 1973, Wright 1973, Markey & Tayler 1973, 1974] and verified via **numerical simulations** [Kiuchi+ 2008, Ciolfi+ 2011,2013, Lasky+ 2011, Sur+ 2021, Cheong+ 2024].
- We study the **stability** of **equilibrium configurations** by performing long numerical simulations.

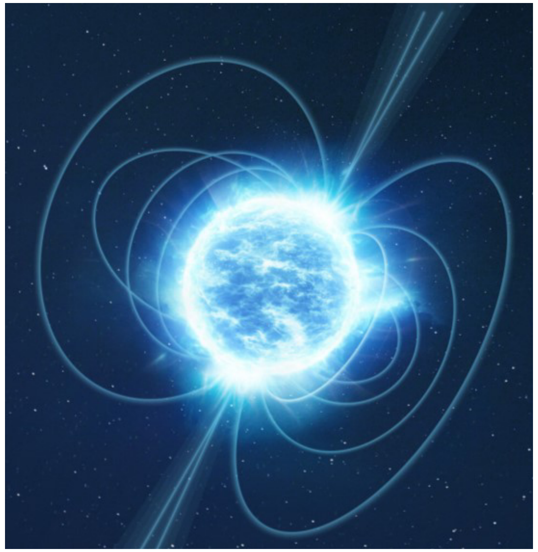
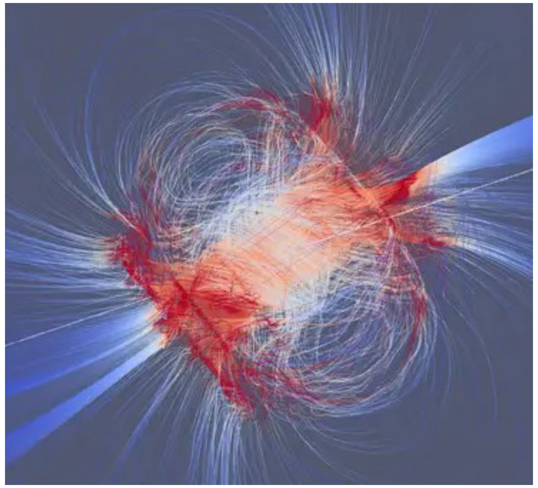


Figure: ESA



Numerical Setup: Athenak

- Open-source C++ code designed for solving **magneto-hydro-dynamics** (MHD) equations with **dynamical spacetime solver** [Stone+ 2024].
- Solves MHD equations with **constrained transport algorithm** [White+ 2016].
- Extension of Athena++ [Stone+ 2020] , rewritten in KOKKOS library [Trott+ 2022]

  → can run on both CPUs and GPUs.
- Excellent **scaling** properties, **portability** and **speedup** efficiency [Fields+ 2024].
- MeshBlock-based **mesh refinement** [Zhu+ 2024].

Publically available at: https://github.com/IAS-Astrophysics/athenak

Simulations

- **Static** TOV star with evolution in **Cowling** approximation.
- Fluid initial data set with gamma law EOS
- 3D domain of ± 80 km and **resolution 156m** [For comparison see Laski+ 2011, Tsokaros+ 2021]
- **Dipole external field** with maximum strength $B = 10^{15}$ G at the surface [Sur+ 2020].
- We investigate internal magnetic field set with **various initial toroidal strengths**.

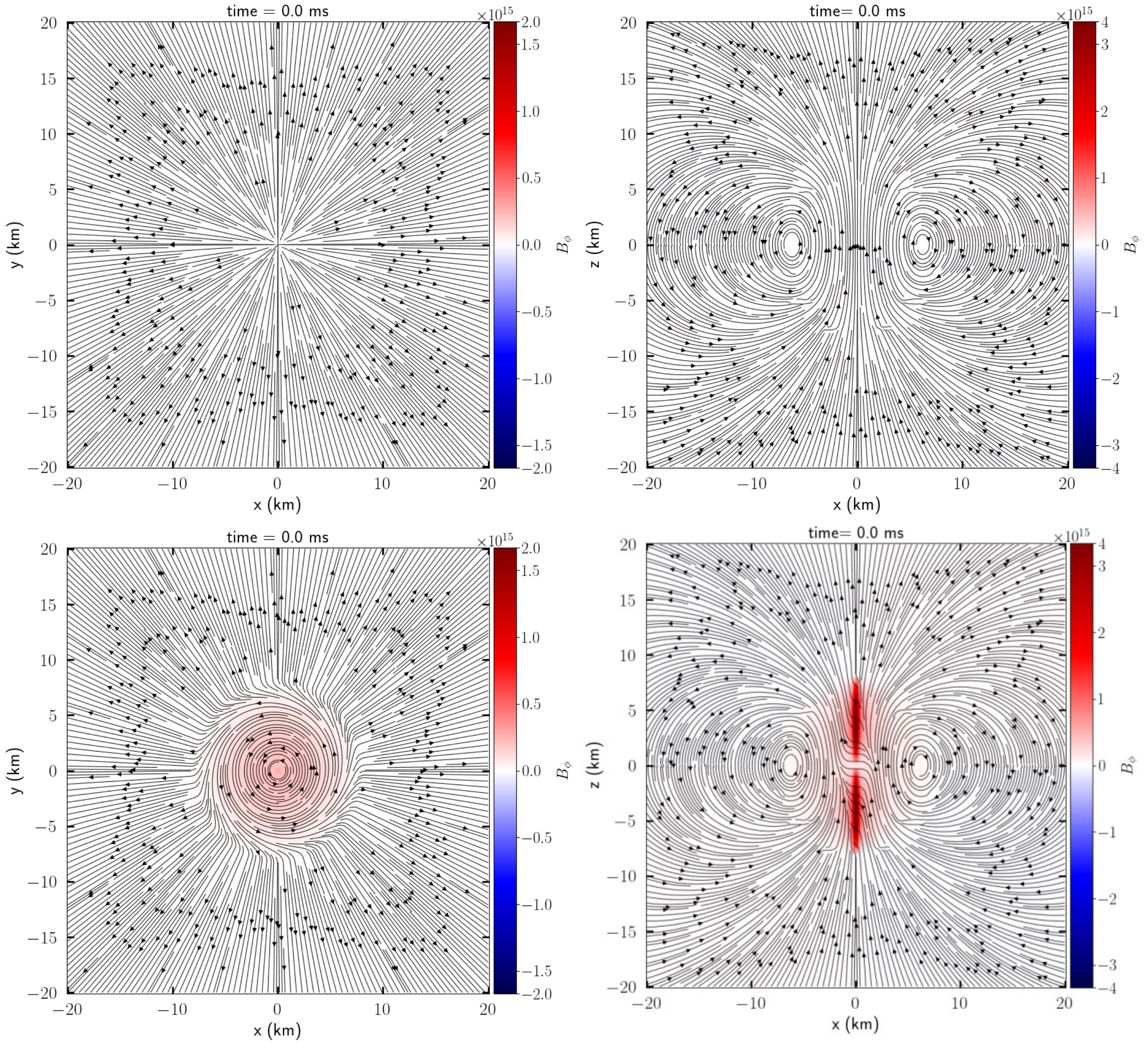
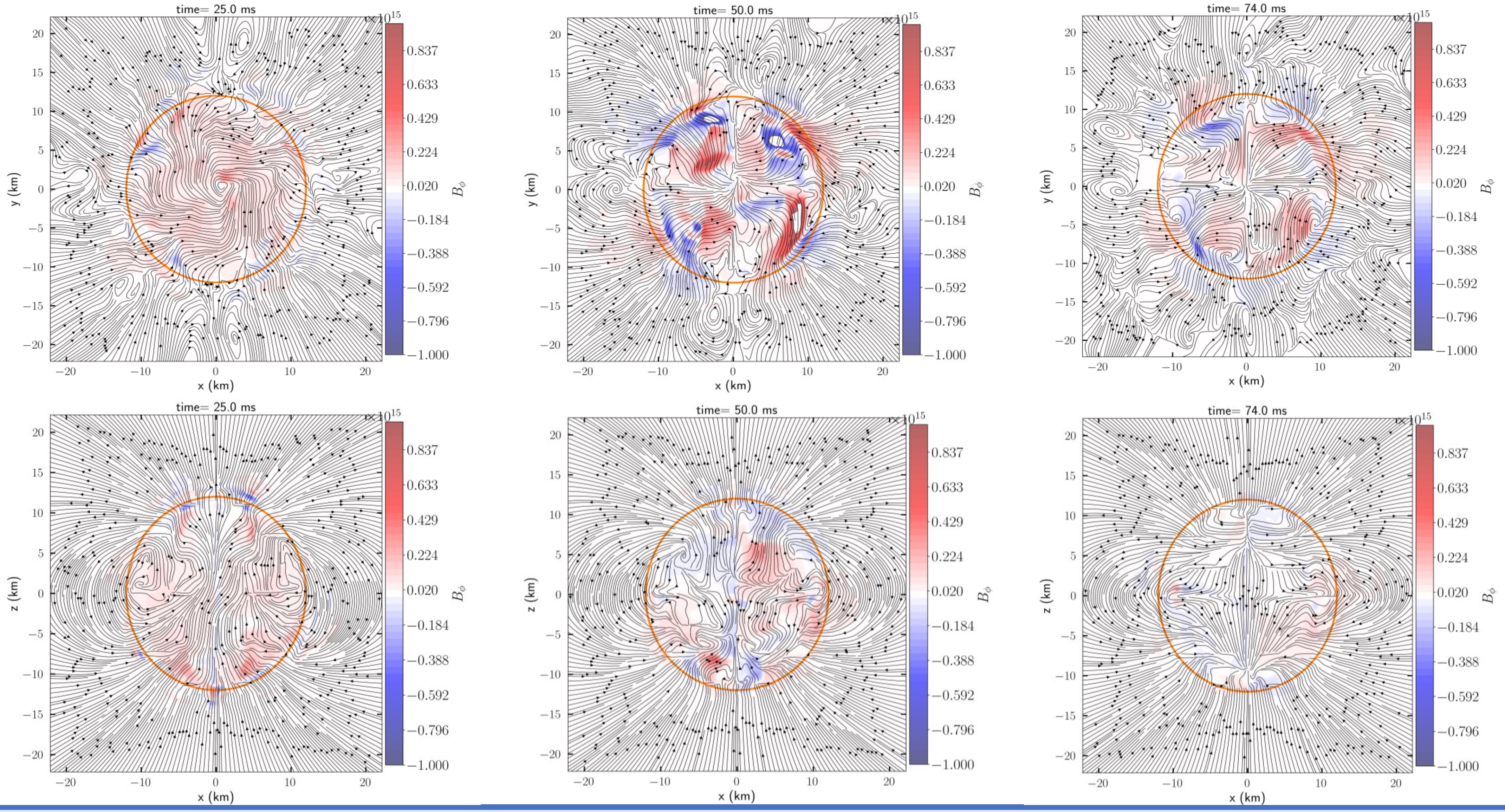
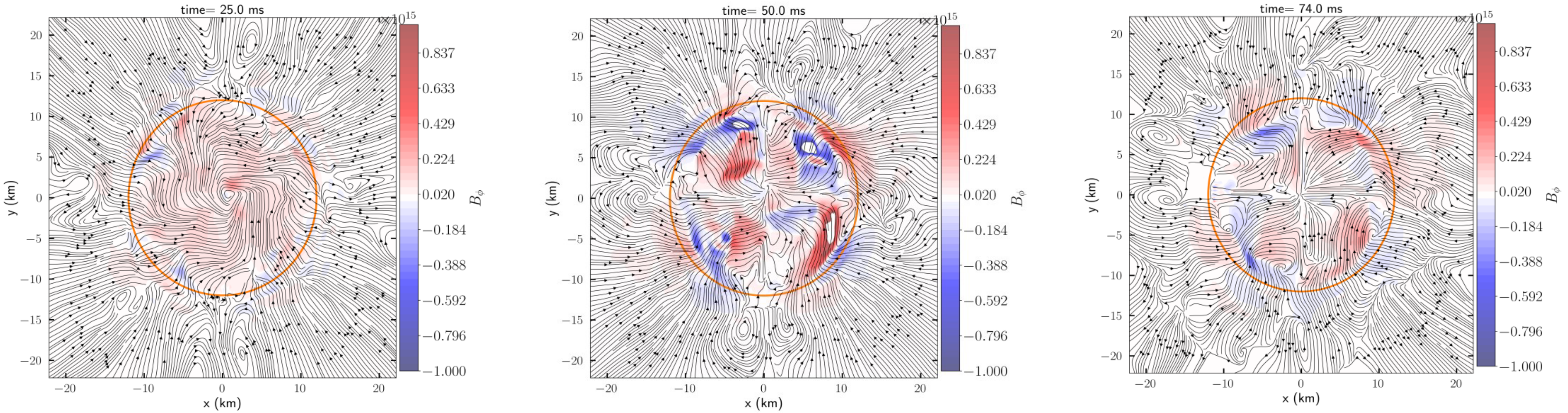


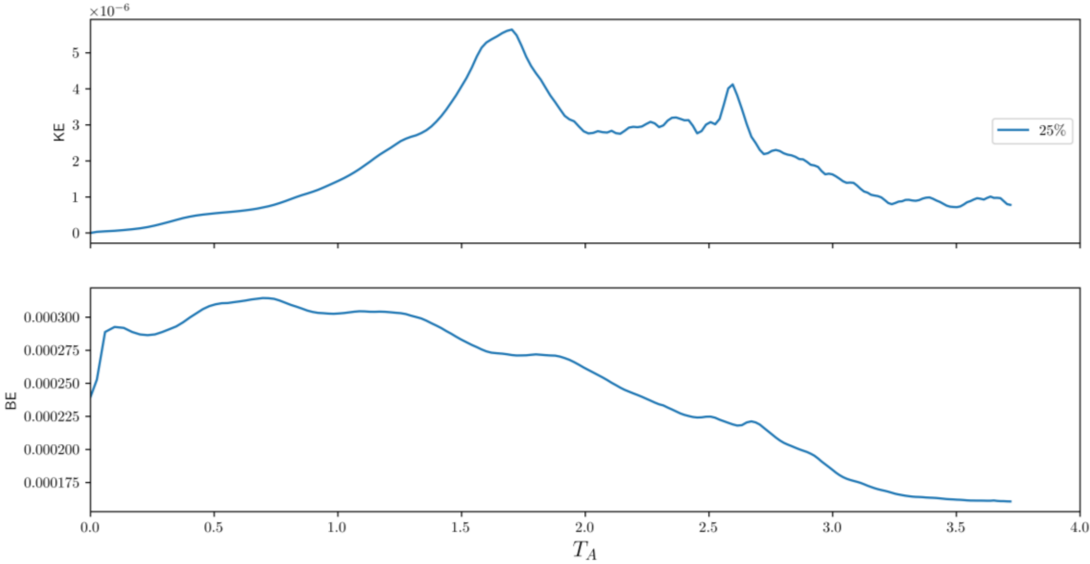
Figure: $t = 0$ setup for $BE_{tor} = 0\%\ BE_{tot}$ and $BE_{tor} = 25\%\ BE_{tot}$

Figures: 2D evolution snapshots for $BE_{tor} = 25\% \, BE_{tot}$



Figures: 2D evolution snapshots for $BE_{tor} = 25\% \, BE_{tot}$





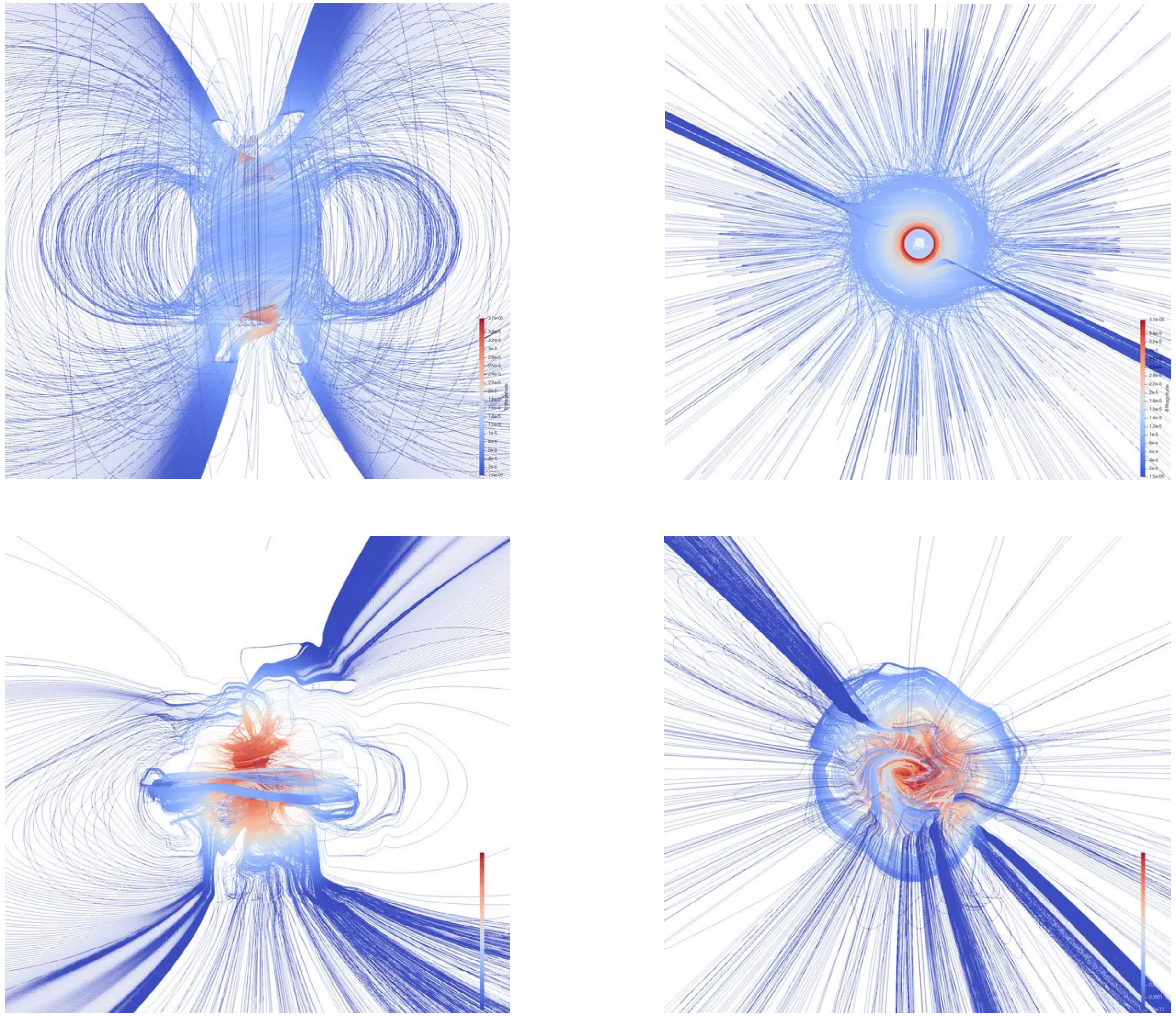
Energies obtained with Eq. B2, B4 from Cook+ 2025

3D evolution snapshots for $BE_{tor} = 80\% \, BE_{tot}$

xy plane | xz plane

t = 0 ms

t = 25 ms



Results:

- **Transient behaviour** after ~ 2 Alfvèn times.
- All configurations reach $\boldsymbol{BE_{tor}} \sim \mathbf{6-18\%}\ \boldsymbol{BE_{tot}}$.
- $\Delta m \sim 10^{-7}, \Delta IE \sim 10^{-5}$.
- Consistent **quasi-equilibrium end state** and correlation with external dipole field.

$$\tau_A = \frac{2R\sqrt{4\pi\langle\rho\rangle}}{\langle B\rangle} \qquad T_A = \int_0^t \frac{dt}{\tau_A(t)}$$

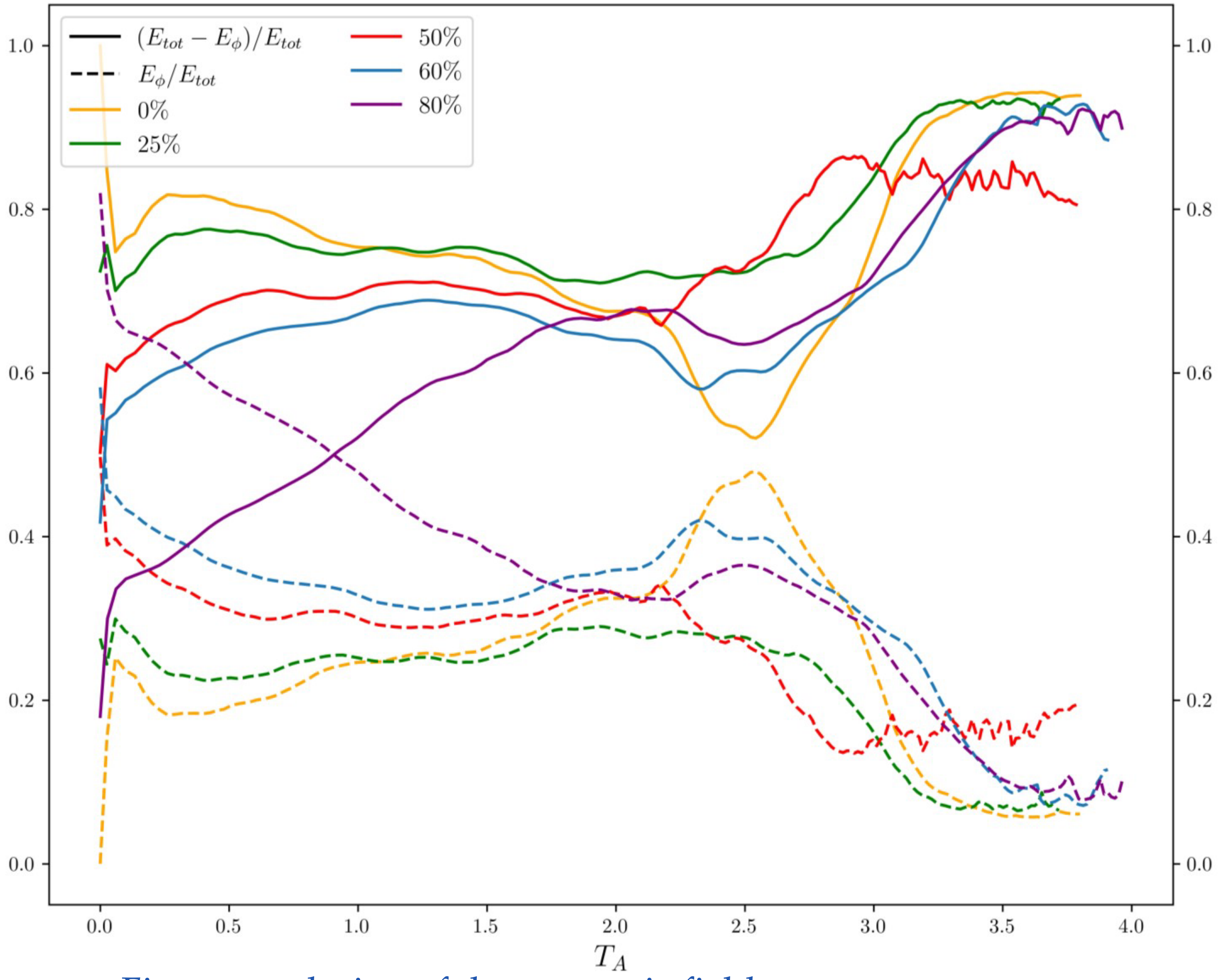


Figure: evolution of the magnetic field energy components (Capobianco+ in prep.)

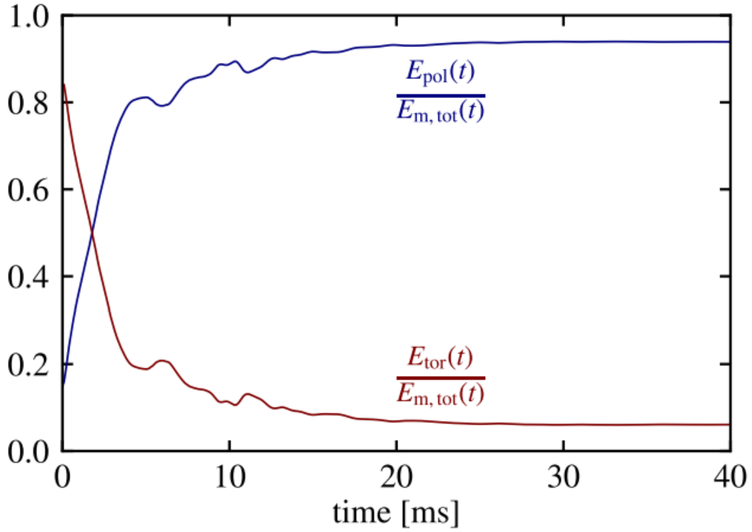


Figure 1 from Sur+ 2020

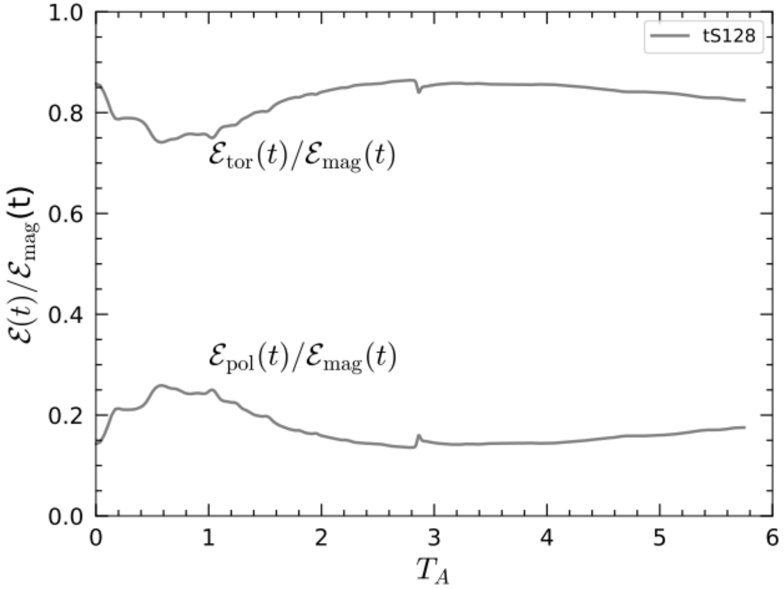


Figure 3 from Sur+ 2022

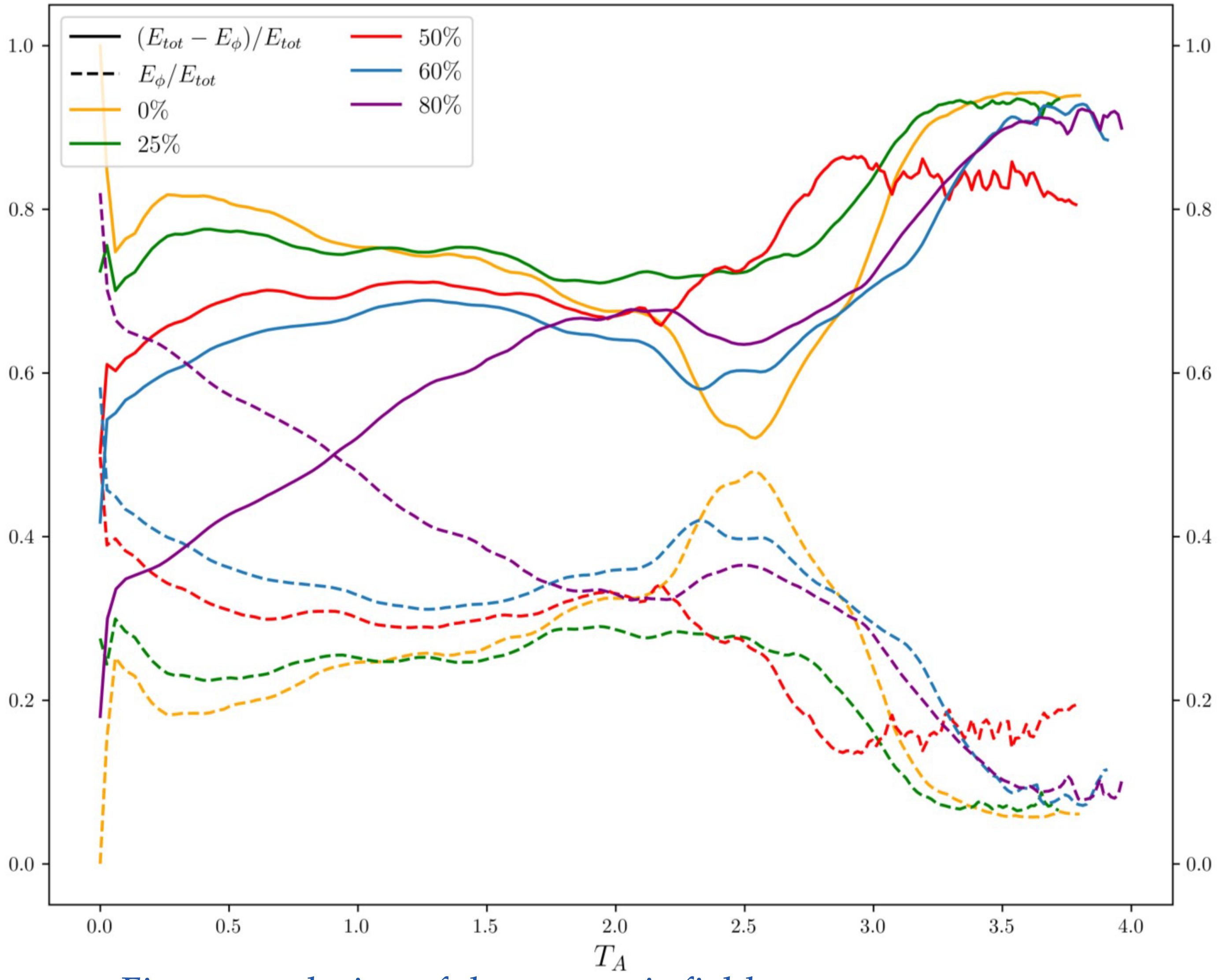


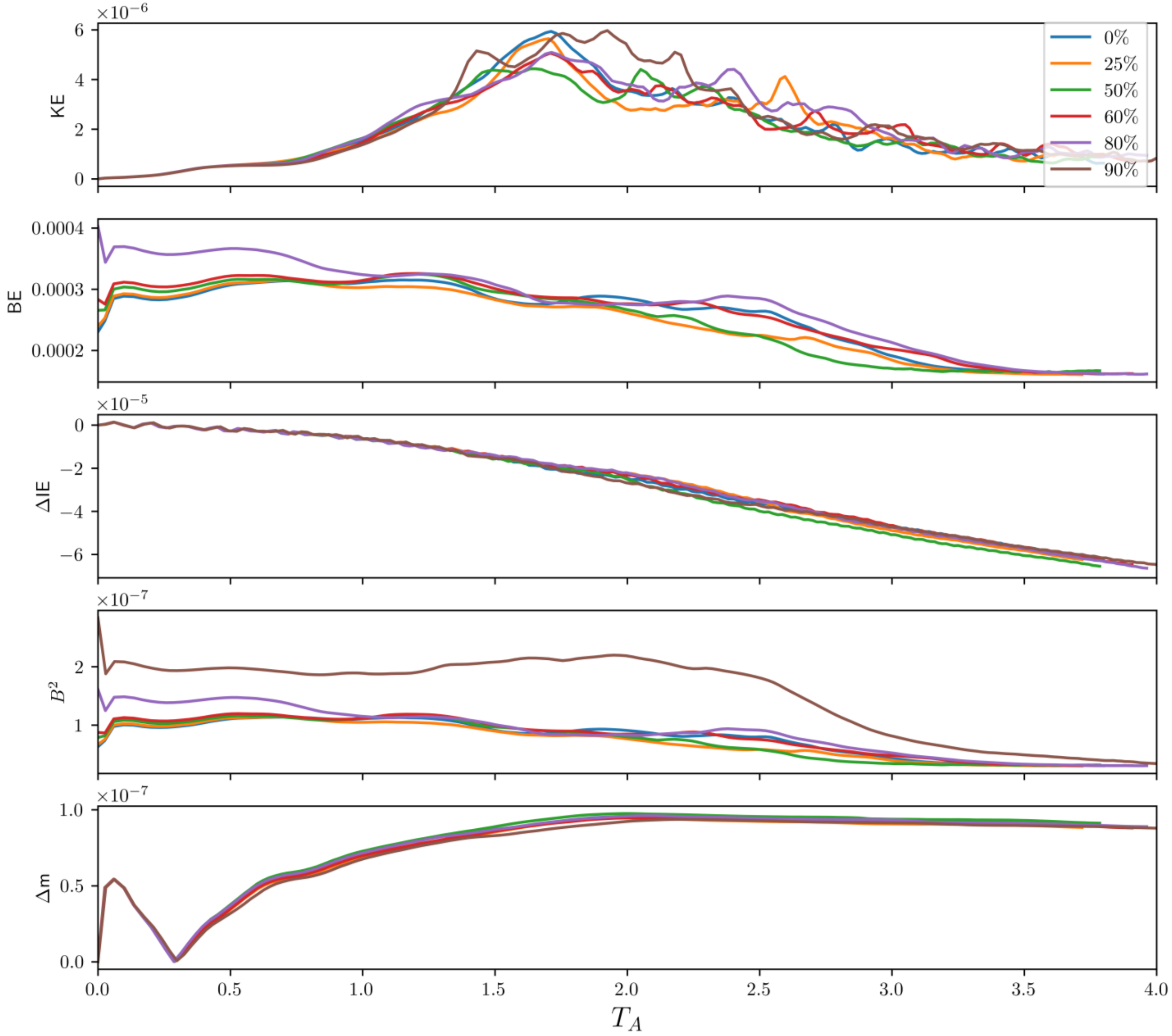
Figure: evolution of the magnetic field energy components (Capobianco+ in prep.)

Conclusions

- **Toroidal magnetic energy decays to 6-18 % of total magnetic energy, independently of the initial BE_{tor} / BE_{pol} ratio**.
- Ongoing **further analysis** to assess effects on **helicity** and **modes**.
- **Longer simulations** (>100ms) needed to verify that this condition is **stable.**
- **Higher resolution** would improve authenticity of results.
- Future work will focus on **rotating NS** and evolution in **full General Relativity**.

Further analysis

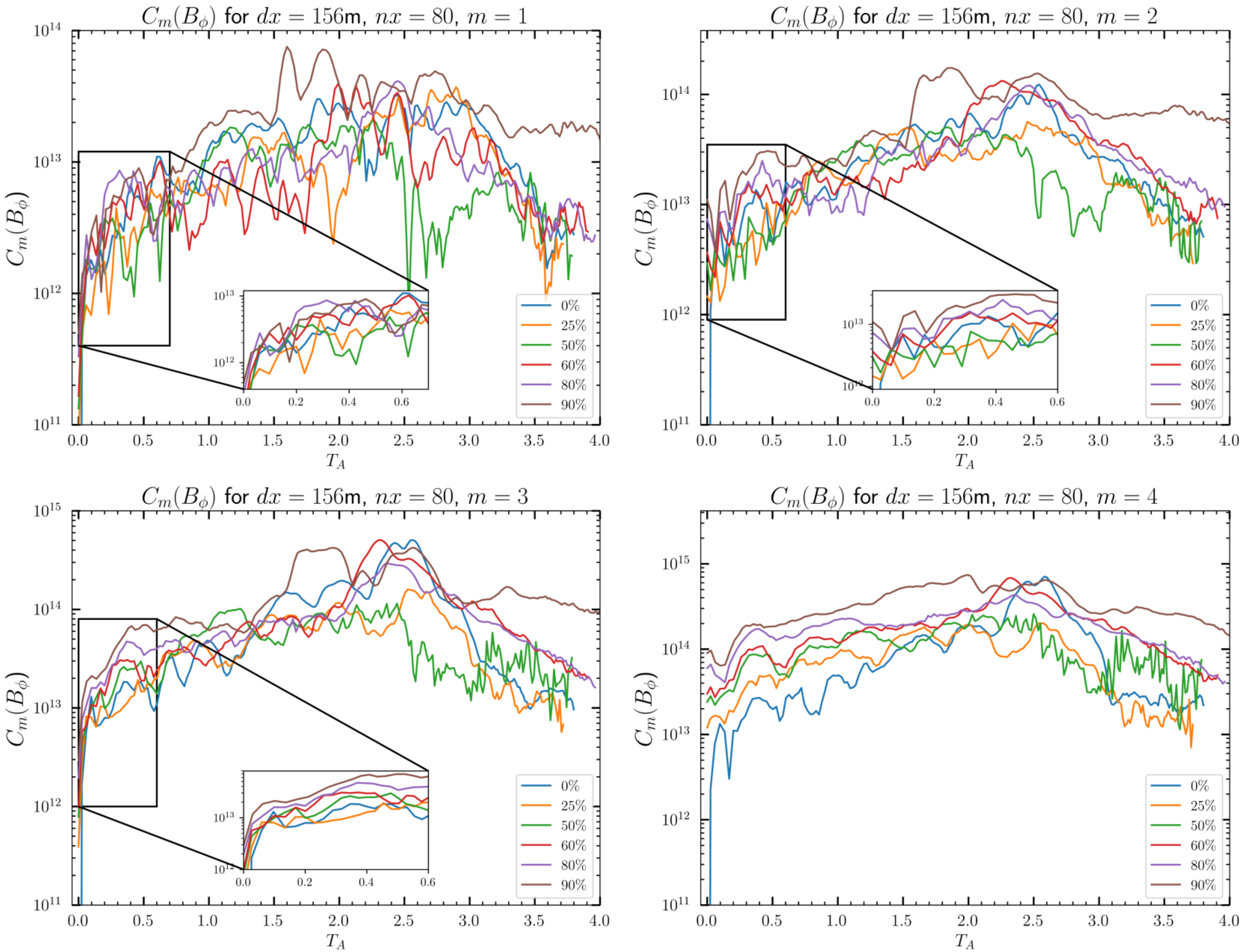
- Comparison of energy and mass conservation behaviour for all configurations



Further analysis

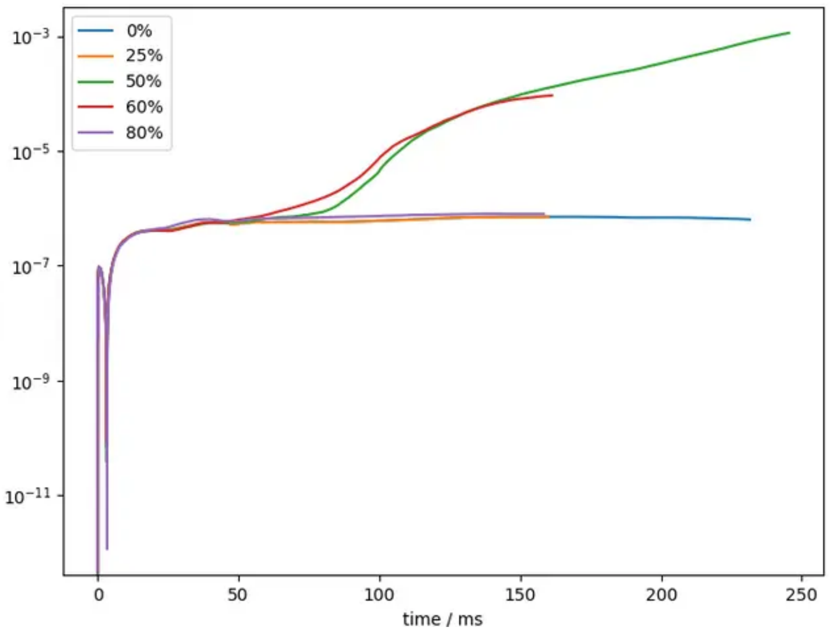
- Fourier modes calculated in star's interior [following Zink+ 2007, Lasky+ 2012]

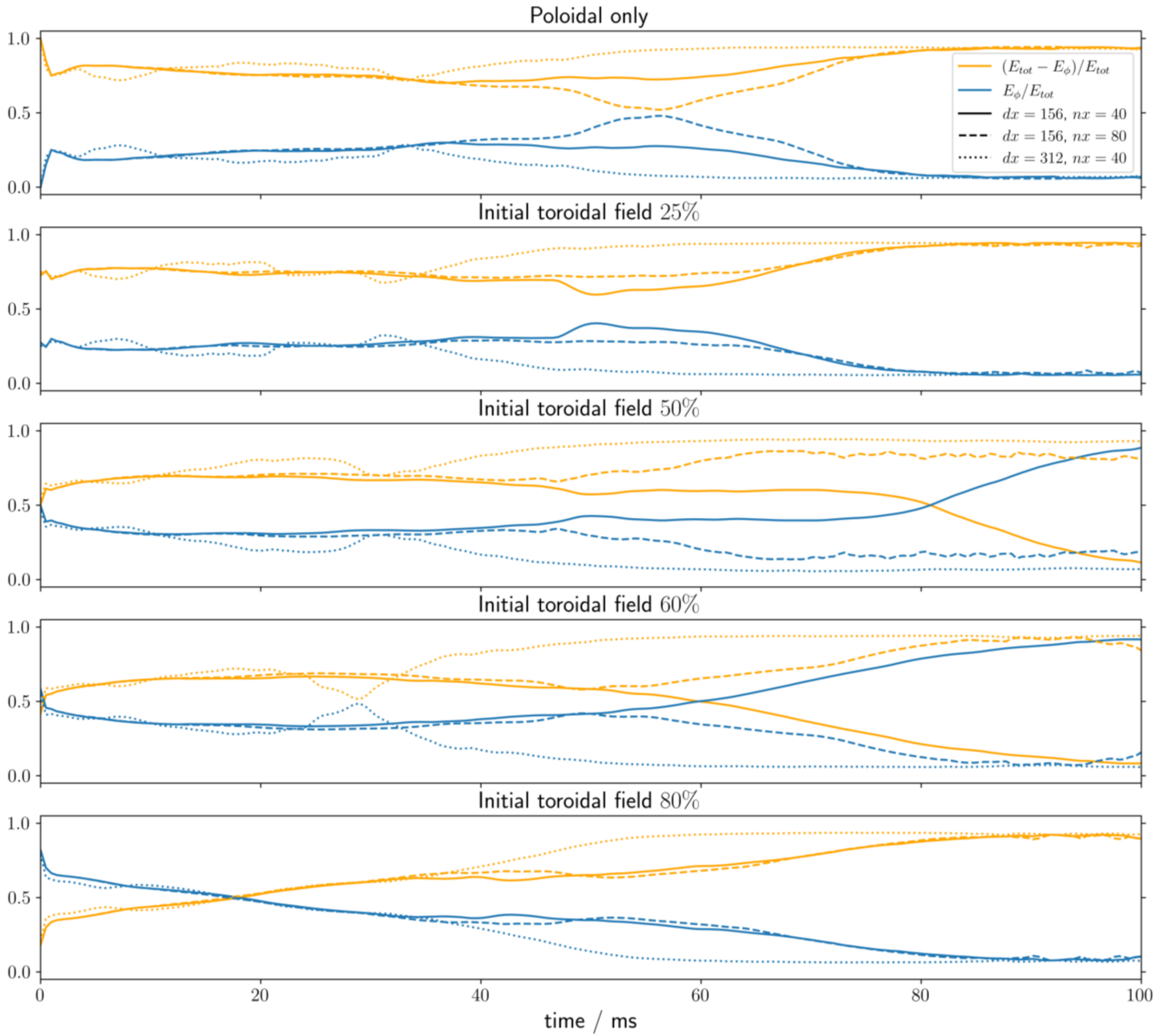
$$C_m = \int_0^{2\pi} B_\phi(\bar{\omega}, \phi, z=0) e^{im\phi} d\phi$$



Resolution & boundaries

Mass violation for domain ±40





External dipole field:

$$B_r = \frac{B_p R^3 \cos\theta}{r^3}$$

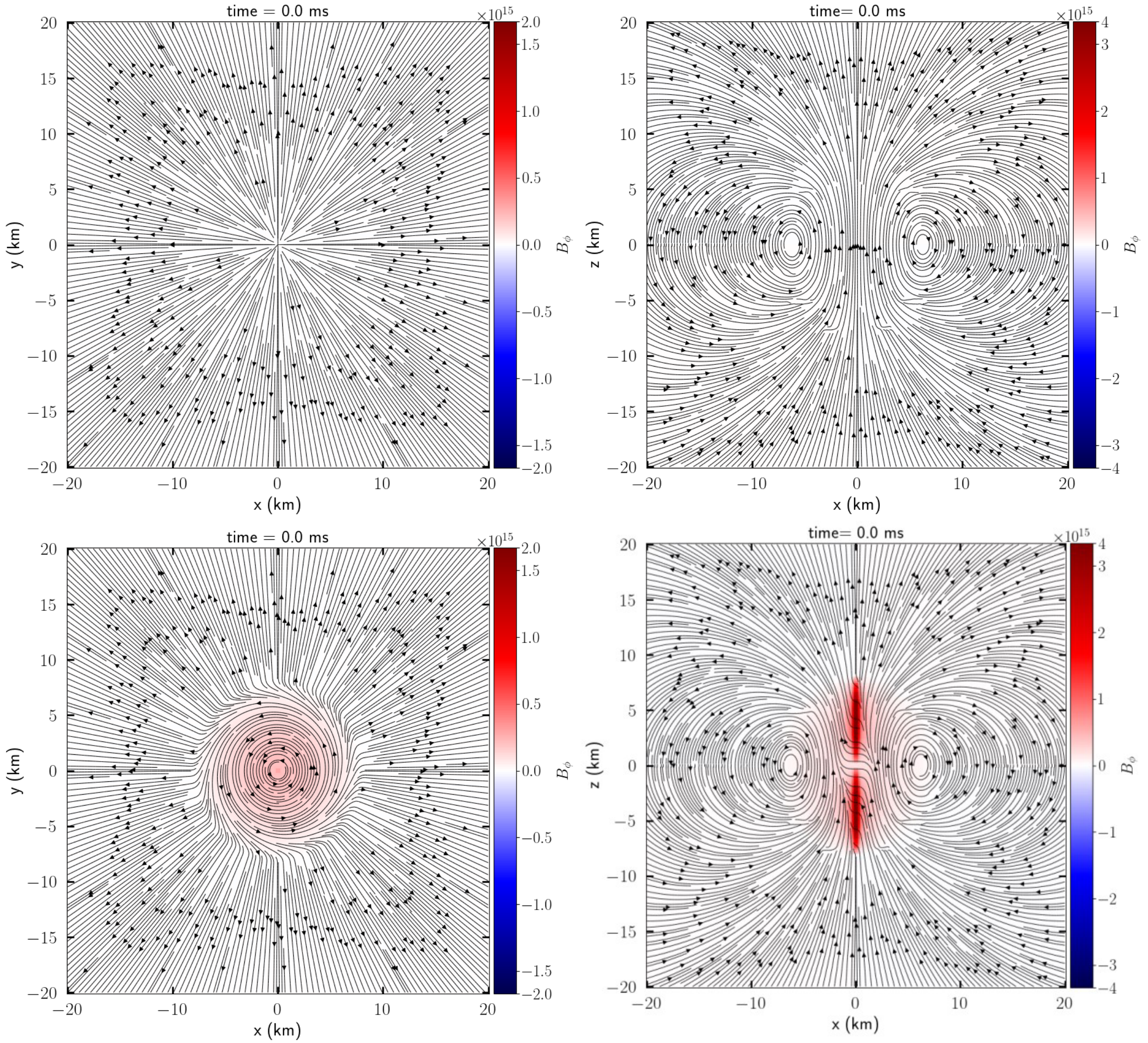
$$B_\theta = \frac{B_p R^3 \sin\theta}{2r^3}$$

Interior configuration:

$$B_r = \frac{B_p \cos\theta}{\pi(\pi^2 - 6)}[y^3 + 3(y^2 - 2)\sin y + 6y\cos y]$$

$$B_\theta = \frac{B_p \sin\theta}{2\pi(\pi^2 - 6)}[-2y^3 + 3(y^2 - 2)(\sin y - y\cos y)]$$

$$B_\phi = B_t \frac{\sin y \sin\phi}{\pi} \qquad \text{with} \qquad y = \frac{\pi r}{R_\star}$$



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